

Visualization Techniques for Computational Mechanics

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ABSTRACT

Tensor data is an integral part of all phases of a modeling and simulation in mechanics of solids and structures. Tensors are used not only for stress and strain, but also for the material properties and behavior; the basic governing equation is $E\varepsilon = \sigma$, where E is the fourth order stiffness tensor, ε is the second order strain tensor, and σ is the second order stress tensor. Critical details of constitutive behavior are rarely visualized since no solid approach has been developed, particularly for the higher order tensors. Here, we propose approaches for graphical representation of stiffness and other constitutive properties, including discrete glyphs to show details, and a continuous graphical representation for fourth order tensors. We discuss the development of a visualization application for students, researchers and practicing engineers.

Keywords: Tensor visualization, higher order tensor, geomechanics, constitutive relation

Index Terms: J.2 [Physical Sciences and Engineering]: Engineering—

1 INTRODUCTION

In this work we will attack the problem of visualizing tensor data as found in mechanics of solids and structures (e.g., civil, mechanical, aeronautical, biomedical, material and other branches of engineering dealing with modeling and simulation of solids and structures). The particular focus of this work is visualization for geomechanics, a branch of civil engineering concerned with the behavior of geomaterials. The specific set of problems concern seismic effects on solids and structures, the so-called soil-foundation-structure interaction problem.

To date, the breadth of visualization technique research for solid (non-fluid) mechanics tensor data has been limited. Meanwhile, the techniques (formulations, algorithms and implementations) for computational modeling and simulation of materials have advanced at rapid pace. While the simulation results are now available, the pictures that will help in understanding those results are absent. Moreover, civil engineering design has evolved in such a way that behavior of critical parts of solids and structures depend on the specific materials and/or components used in an object. Finally, civil engineering materials are known to be loading-history dependent; that is, their behavior is a function of their previous loading history. For example, soils that have been previously compressed will be much stiffer than those not compressed in their loading history. This dimension adds an additional challenge to presenting the most interesting and important parts of the data in a way that is useful and understandable to a wide audience of practicing engineers, researchers, and students.

2 MAIN CONTRIBUTIONS

There are several contributions contained in this thesis proposal. First, we provide an open source visualization application that

works in tandem with the OpenSees earthquake engineering simulator [7]. We plan to invite the earthquake engineering community to participate in user studies and to solicit their feedback and suggestions for features. Secondly we plan to address a specific need for the visualization of solids and structures: a representation for the equation describing constitutive relation (CR) and other details of constitutive behavior.

3 METHODOLOGY/ PLAN OF RESEARCH

We plan to attack the problem by first providing a general application to allow exploration of a simulation domain. Secondly, we plan to provide a means for the user to focus in on a specific element within a domain and provide new techniques to show the critical details at the location. The hope is that we will come to understand what is important to observe at both levels and so help future users to more easily find the “good and the bad stuff”.

For this exploratory phase we are using Shneiderman’s well known three level approach from information visualization: overview, zoom and filter, details-on-demand [8]. The first two levels are already well underway. We are writing a visualization application that rides directly onto the object-oriented OpenSees finite element simulator. The simulator provides immediately all geometry as well as associated scalar and tensor data. At the overview level we provide solid and mesh visualizations, (including meshes deformed as the simulation progresses) as well as the plane-in-the-box glyph that we presented at last year’s IEEE Visualization [6]. For zoom and filter we provide several approaches. First, the user may adjust the upper and lower threshold for a number of classical scalars (e.g., isotropy, magnitude of deformation, etc.). A new metric for constitutive behavior will be included in this group. The threshold range may also be inverted; we showed last year that this is an extremely useful feature [6]. Additionally, we already automatically generate a list of the component types in the simulation and the user can turn on or off the types she would like to see. For instance, the user may want to see just soil, just beams, etc. We plan to provide the same mechanism for selection by material type (e.g., sand versus clay). Finally we have built the object-oriented framework so that individual finite elements can be assigned individual visualization techniques (it still remains to make this available through the user interface). The open source code is available through NEESforge [5] and is free for non-commercial and academic use.

A very exciting part of the research is at the level of details-on-demand. We plan to provide the aforementioned snowflake glyph for the CR and stress. We will be comparing its use to the glyph described in by Kriz [3] and Hlawitschka [1]. This however, is only the first step toward a continuous representation.

We also plan to show other details of constitutive behavior at the integration points, in advance of the continuous representation. Important features include yield surfaces, plastic flow directions, internal variables, state of stress, and stiffness tensor.

4 YIELD SURFACE

The yield surface is defined in a space spanned by principal stresses σ_1 , σ_2 , and σ_3 (for isotropic behavior only principal stresses are enough) as a border between the elastic and elastic-plastic region. At the yield surface, a material begins to deform plastically, causing permanent deformation. Being able to observe a yield surface

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change over time would help engineers verify that their material model is correct and also give insight into the state of materials during a simulation. Additionally, it provides a platform to visualize numerical algorithms (for example, numerical integration of constitutive, elastic-plastic equations), showing the predictor and the corrector steps in relation to the yield surface [2].

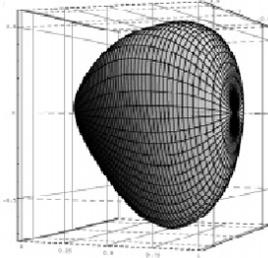


Figure 1: Yield surface in principal stress space [2].

A yield surface is, by definition, a convex surface. It is often ellipsoidal, and could easily be visualized as a single glyph. Some materials are modeled with multiple nested yield surfaces and these could be rendered together in a single space, with each surface at reduced opacity. A time-varying yield surface could be animated by gradually reducing the opacity of the old yield surface while increasing the opacity of the new.

5 FOURTH ORDER CONSTITUTION TENSOR

The constitutive relation represents the relationship between applied stress and the resultant strain (deformation) that a material undergoes. The constitution of the material is described by a fourth order symmetric tensor. The current technique for visualizing fourth order tensors is a glyph (similar to a Reynolds glyph), as described in Kriz [3] and also Hlawitschka [1]. However, interpreting the meaning of a complicated surface is difficult. We propose a new glyph we call the snowflake glyph where the branches of the snowflake represent associated surfaces and vectors on an infinitesimally small cube that the matrix values represent. Each branch of the snowflake points outward if the matrix entry's absolute value is greater than one (representing stretch) and inward for an absolute value less than one (shrinkage). Overlaying the stress snowflake on the CR should allow prediction of the resultant strain. In short we will provide a general method for visualizing matrix multiplication of matrices differing by two orders.

5.1 Continuous Representation

The glyph approach may be sufficient for the "snapshot" of a homogeneous region. In order to show the larger picture we are working toward a similarity metric to decide if two neighboring elements CR's are sufficiently similar to be classified as the same region. The similarity measure is based on the main constitutive equation:

$$\sigma_{ij} = E_{ijkl}\epsilon_{kl} \quad (1)$$

The basic idea for classifying material regions by stiffness matrix E_{ijkl} is to assign matrix entries letters of the alphabet and sort them entries (as originally proposed by Musgrave [4]). This simple example shows classification of a 2×2 tensor (in reality we would sort the entire 9×9 stiffness tensor).

$$\begin{aligned} S_{xx} &\equiv A \\ S_{yy} &\equiv B \\ S_{xy} &\equiv C \\ S_{yx} &\equiv D \end{aligned}$$

There will be an order (some permutation) of the letters when sorted by their respective values, for example, $B \geq A \geq C \geq D$. We

can save the sorted entries as a string $BACD$. The basic idea is then that we can compare two materials based (at the coarsest level) on their greatest common substrings.

We need some new extra rules to correctly differentiate material properties, since we are basing our classification on the effects of a matrix multiply with indefinite values.

- Handle the case when 2 adjacent letters' values are *nearly equal* by accepting a reverse of the substring, For instance, CD matches CD or DC . For substrings of length three or more with nearly equal values, any permutation of the substring must match. The reason for the criteria of near equality is due to the use of floating point numbers.
- Both cases must have the same *sign*, because a negative sign switches stress between states of tension and compression, while a positive sign has no effects.
- Both cases must have an absolute value either
 - less than 1 (causing stress to shrink)
 - greater than 1 (cause stress to grow)

If all these criteria match, along with a string match, we have a "similar" material. Both materials will cause the same types of effects for a given strain. Therefore, they can be part of the same region. For finer grained similarity, we could look at sum of squares of difference of letter-by-letter value, or other similarity metrics.

With such a similarity measure, we may use a difference operator between two regions, or other operators such as gradients. This would allow us to study when homogeneous regions collapse and merge, and what leads to these events, as well as providing a continuous graphical representation of constitutive relations.

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